Product of Sums Investigation

When completing an investigation, organisation of your results is paramount as this allows you to see patterns more easily.

In order to ascertain a pattern, sometimes, it is a good idea to plot a graph to help you; sometimes organising your data into a table and sometimes looking for a model such as an equation helps you to see what is going on.

The number 24 has the factors {1, 2, 3, 4, 6, 8, 12, 24}.

However, there are also many ways of making 24 using addition. $\{0+24, 1+23, 2+22, ..., 11+13, 12+12\}$

In addition to this, we can also make 4 by using mixed numbers such as $8\frac{2}{3} + 15\frac{1}{3}$ etc.

Challenge One

Find which two numbers, when you split 24, will yield the greatest product when you multiply the numbers together.

eg
$$15 + 9 = 24$$
. $15 \times 9 = 135$.

$$8\frac{2}{3} + 15\frac{1}{3}$$
. $8\frac{2}{3} \times 15\frac{1}{3} = 134\frac{1}{3}$

Challenge Two

If the numbers were split into three parts eg 8 + 2 + 14 = 24. $8 \times 2 \times 14 = 224$.

What is the biggest number you can make in this way when you split 24 into three numbers?

Is this bigger or smaller than two numbers?

Challenge Three

Using the same methodology as challenges one and two, you can split 24 into anything up to 24 numbers using integers. For this challenge, stick to using integers alone and see if you can determine what number of numbers yields the highest product and how best to determine the highest product of that number of numbers.

Challenge Four

Choose an odd number and complete the same investigation but also including basic fractions such as quarters, halves and three quarters to see if this alters anything.